The Music of the Spheres

Daniel Litt

Stanford University

January 9, 2014

Exercises

- Suppose the only celestial bodies were the sun and the earth. What would the difference be between "the sun going around the earth" and "the earth going around the sun"?

Exercises

- Suppose the only celestial bodies were the sun and the earth. What would the difference be between "the sun going around the earth" and "the earth going around the sun"?
- In the universe as we know it, what is the difference between "the sun going around the earth" and "the earth going around the sun"?

Exercises

- Suppose the only celestial bodies were the sun and the earth. What would the difference be between "the sun going around the earth" and "the earth going around the sun"?
- In the universe as we know it, what is the difference between "the sun going around the earth" and "the earth going around the sun"?
- What features of the night sky can we observe with the naked eye?

Exercises

- Suppose the only celestial bodies were the sun and the earth. What would the difference be between "the sun going around the earth" and "the earth going around the sun"?
- In the universe as we know it, what is the difference between "the sun going around the earth" and "the earth going around the sun"?
- What features of the night sky can we observe with the naked eye?
- What path does Mars make in the night sky? (Draw it.)

What Does the Sky Look Like?

- $\sim$ 5600 stars visible in a dark sky (in a modern city, more like 200 - 500).

What Does the Sky Look Like?

- $\sim$ 5600 stars visible in a dark sky (in a modern city, more like 200 - 500).
- Distinguished features:

What Does the Sky Look Like?

- $\sim$ 5600 stars visible in a dark sky (in a modern city, more like 200 - 500).
- Distinguished features:
  - The sun and the moon,

What Does the Sky Look Like?

- $\sim$ 5600 stars visible in a dark sky (in a modern city, more like 200 - 500).
- Distinguished features:
  - The sun and the moon,
  - Certain bright lights (planets) move in apparently erratic patterns, within a narrow strip: Mercury, Venus, Mars, Jupiter, and Saturn

What Does the Sky Look Like?

- $\sim$ 5600 stars visible in a dark sky (in a modern city, more like 200 - 500).
- Distinguished features:
  - The sun and the moon,
  - Certain bright lights (planets) move in apparently erratic patterns, within a narrow strip: Mercury, Venus, Mars, Jupiter, and Saturn
  - There are some differences in the night sky depending on one's location and the time of year.

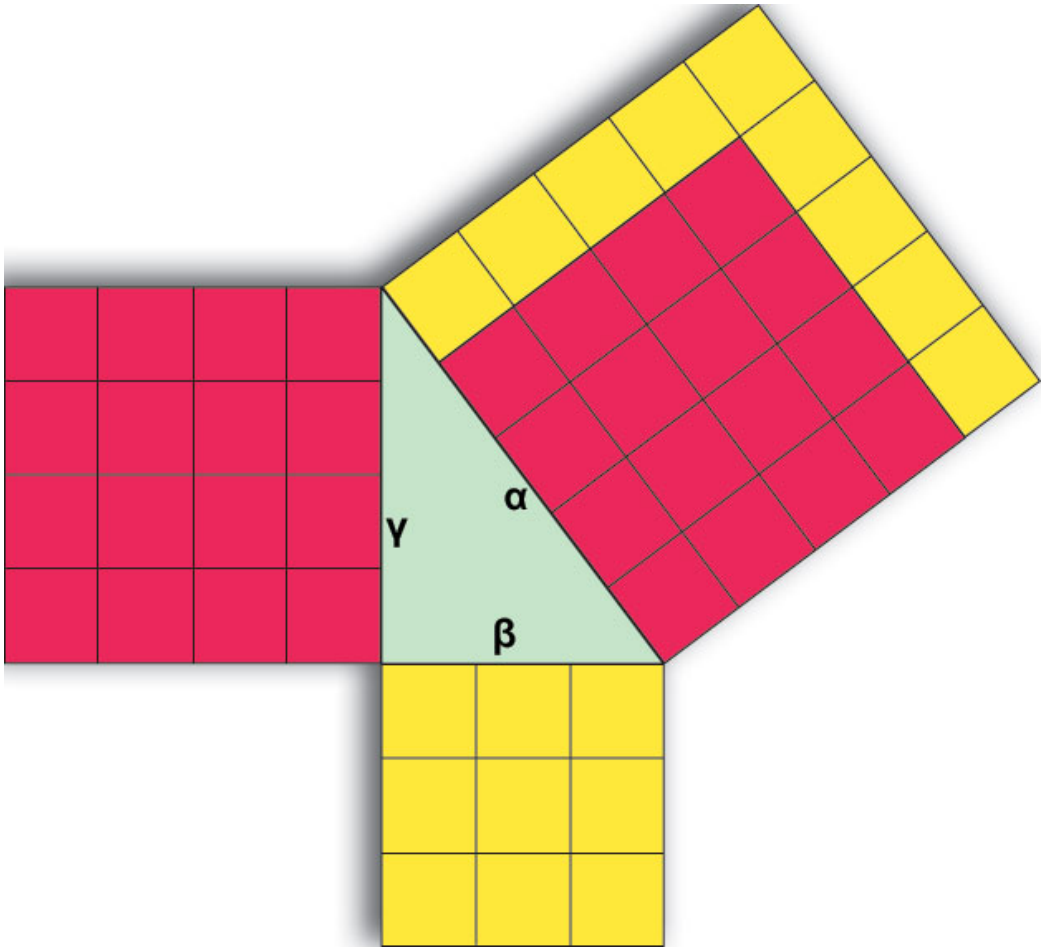
What Does the Sky Look Like?

Retrograde Motion of Mars

The Pythagoreans

Pythagoreans Celebrate Sunrise, Bronnikov

The Pythagoreans



The Pythagorean Theorem

The First Great Discovery

The Pythagoreans discovered: we can hear rational numbers.

The First Great Discovery

The Pythagoreans discovered: we can hear rational numbers.

$$\frac{p}{q},\ p \text{ and } q \text{ whole numbers.}$$

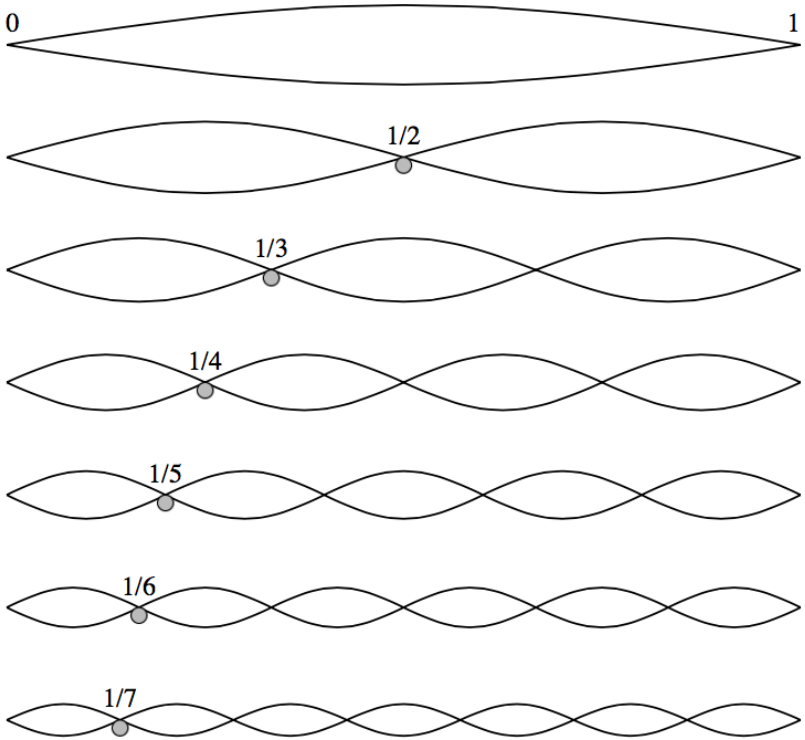
The First Great Discovery

The Pythagoreans discovered: we can hear rational numbers.

$$\frac{p}{q},\ p \text{ and } q \text{ whole numbers.}$$

Suppose one has two strings made from the same material of length x and y. Then plucking both sounds "harmonious" only if $\frac{x}{y} = \frac{p}{q}$ is a rational number with p, q small (say, less than 7 or so).

The First Great Discovery

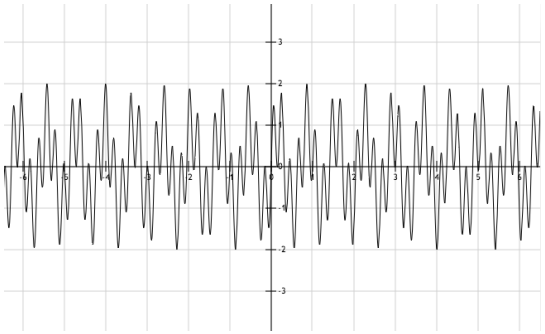


Harmonics (the harmonic series)

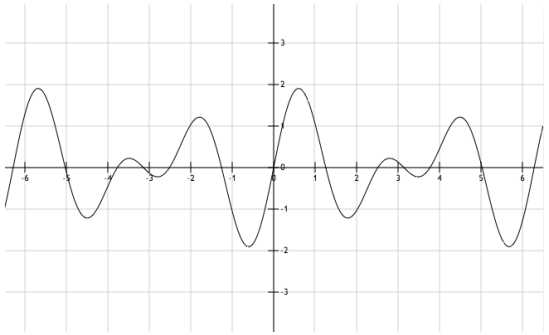
The First Great Discovery

Any sound can be made from these basic sounds!

The First Great Discovery

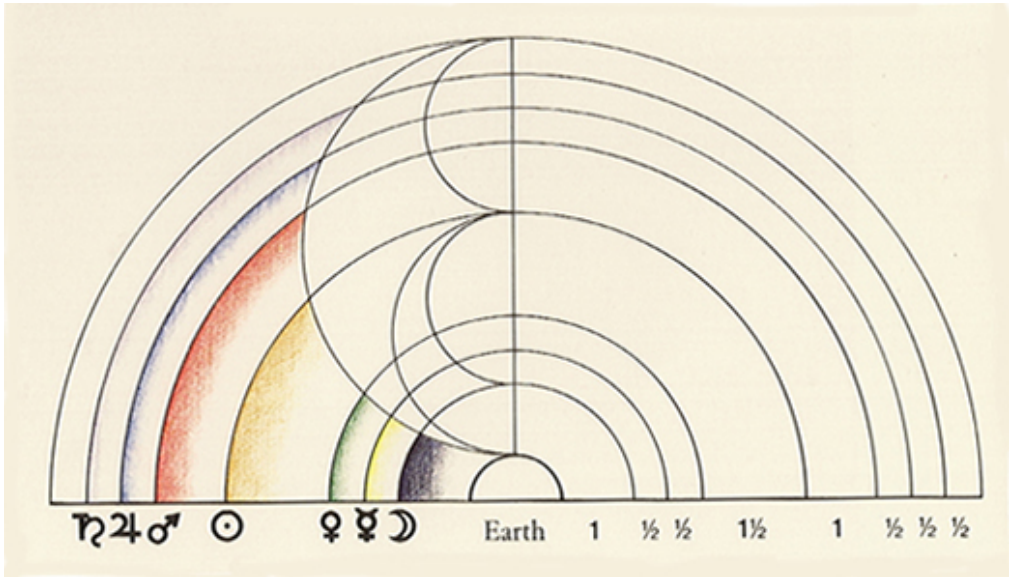


An Inharmonious Note



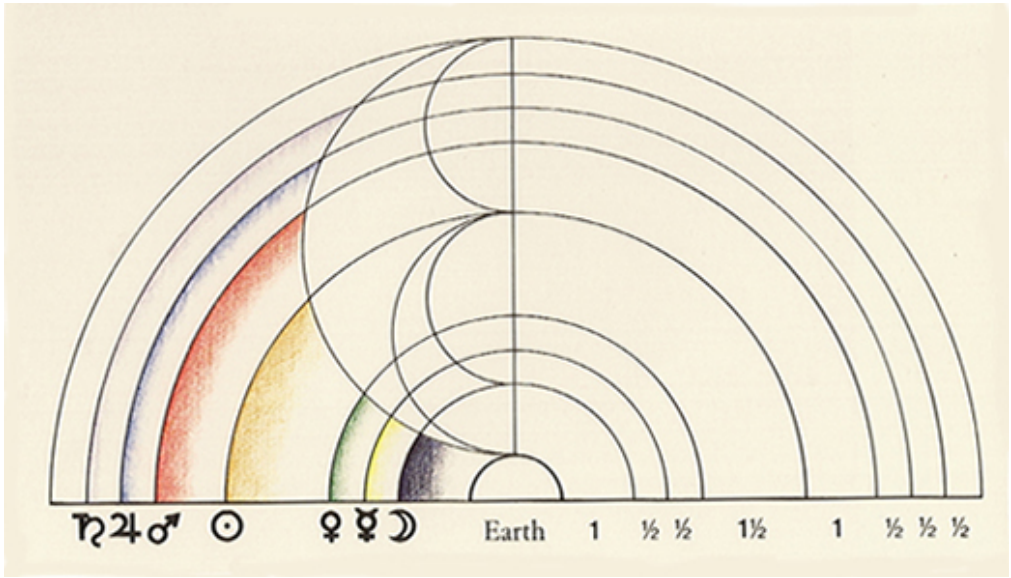
A Fifth

Back to Cosmology



The Pythagorean Scale

Back to Cosmology



The Pythagorean Scale

According to Pythagoras, the planets were arranged according the most harmonious scale. This was the music of the spheres–and the Master, Pythagoras, could hear it.

Tragedy

All was rational and harmonious, until:

Tragedy

All was rational and harmonious, until:

$$\sqrt{2}$$

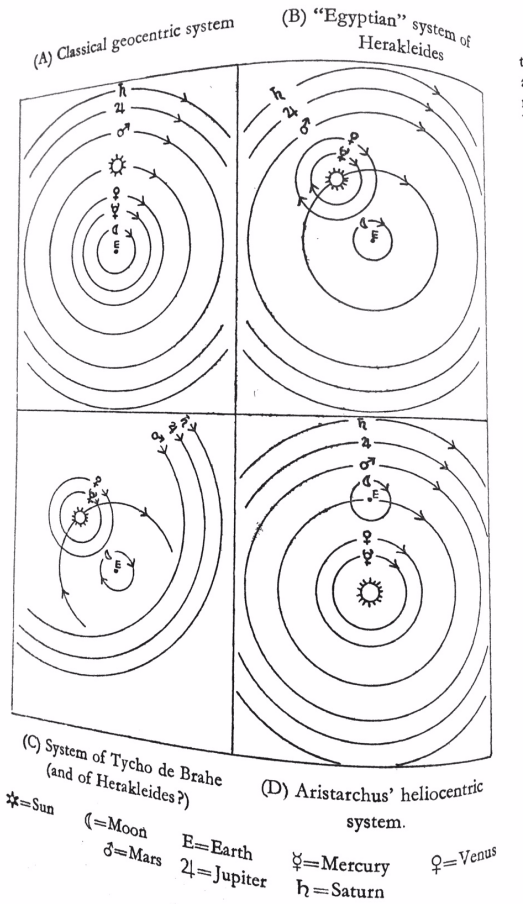
Tragedy

All was rational and harmonious, until:

$$\sqrt{2}$$

"It is told that those who first brought out the irrational from concealment into the open priced in shipwreck, to a man. For the unutterable and the formless must needs be concealed. And those who uncovered and touched this image of life were instantly destroyed and shall remain forever exposed to the play of the eternal waves." –Proclos

Other Greek Theories



Other Greek Theories

A Problem

- But the Greeks knew: the opposite side of the world was always shrouded in darkness.

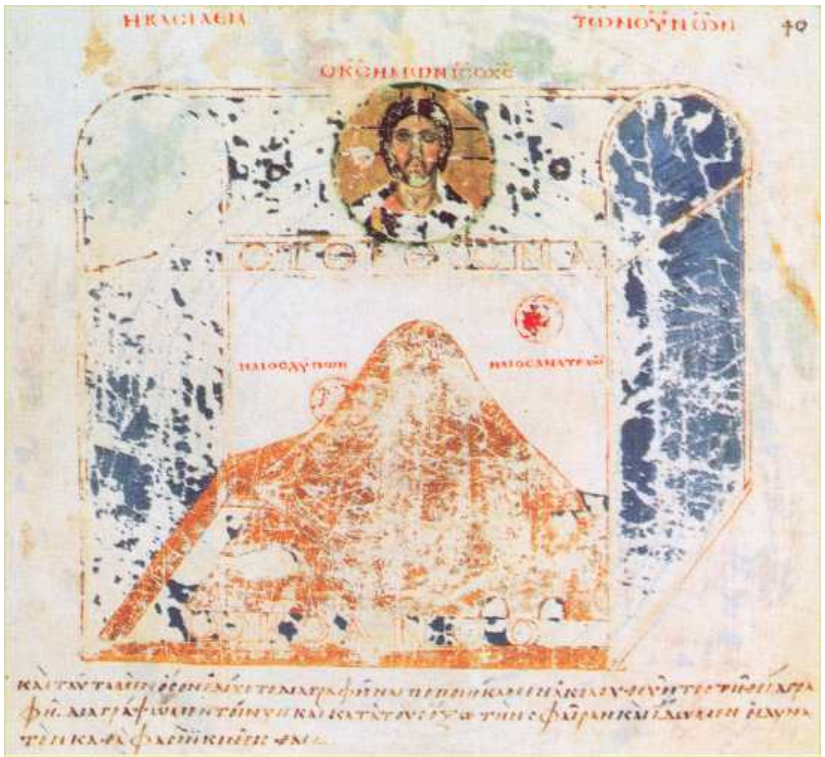
A Problem

- But the Greeks knew: the opposite side of the world was always shrouded in darkness.
- Possible explanation (Philolaus): a "Central Fire" about which the Earth orbited, and a counter-Earth, to block out its light.

Cosmas and the Medievalists

Cosmas's Theory

Cosmas and the Medievalists



Cosmas's Theory

Explains why rivers run faster towards the South-East.

Plato, Aristotle, Ptolemy, and Copernicus

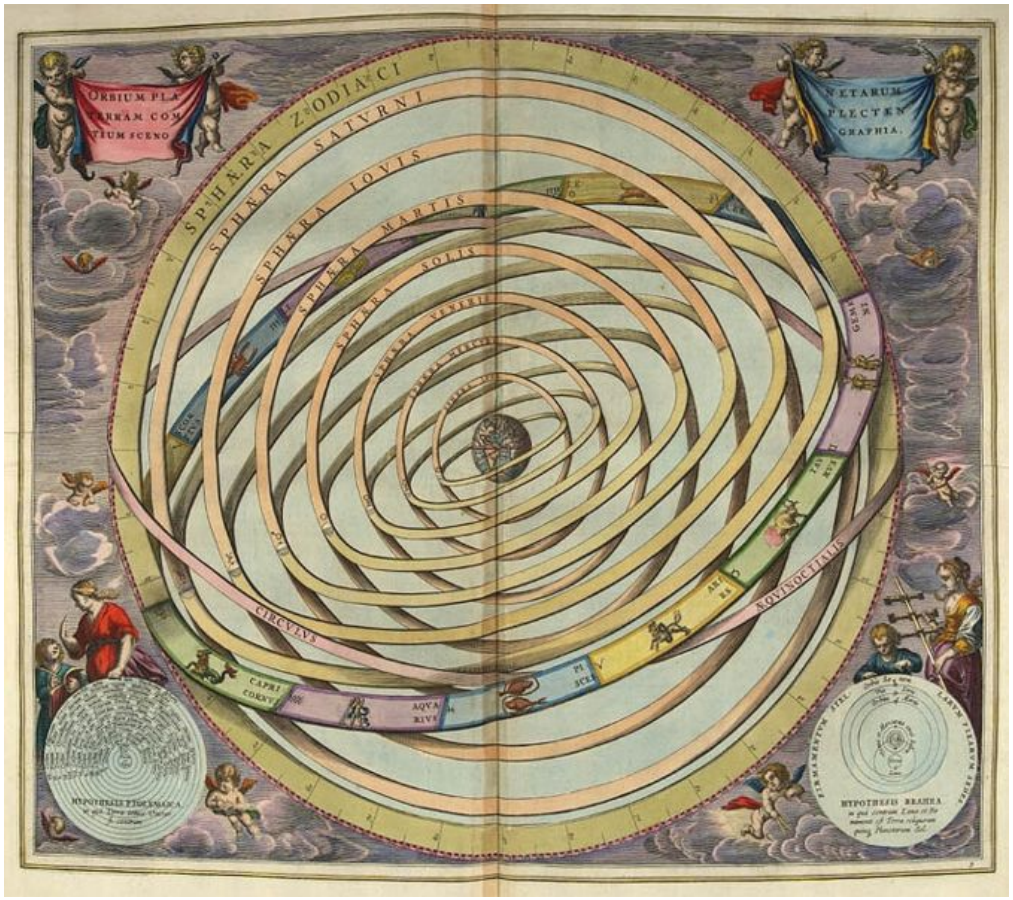
Plato

Plato, Aristotle, Ptolemy, and Copernicus

Plato

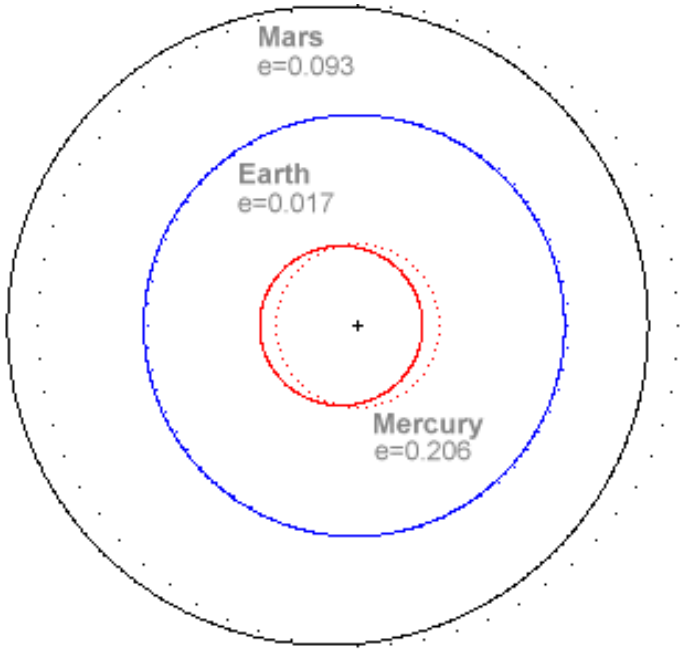
Plato decreed that all heavenly bodies move in perfect circles

The Aristotelian Conception



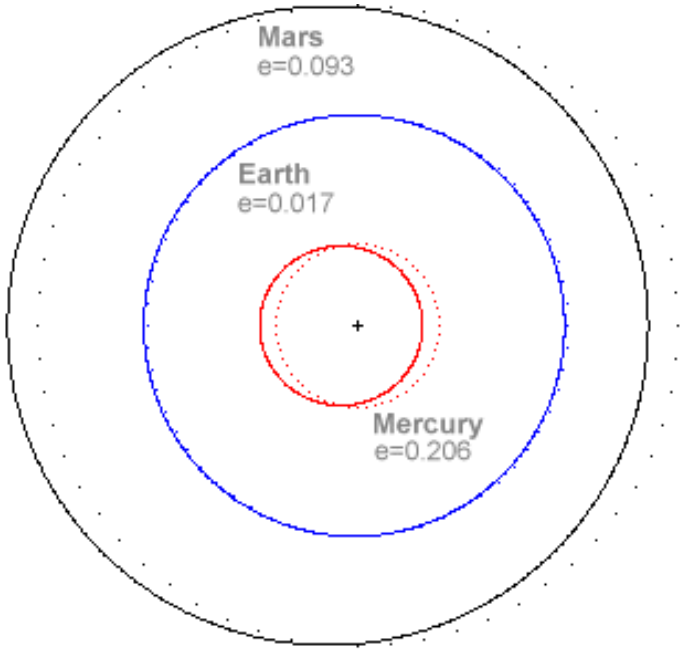
The Crystalline Spheres

Some Problems



The Orbits of Mercury, Earth, and Mars

Some Problems



The Orbits of Mercury, Earth, and Mars

Not to mention: the planets don't orbit around the Earth!

Ptolemy

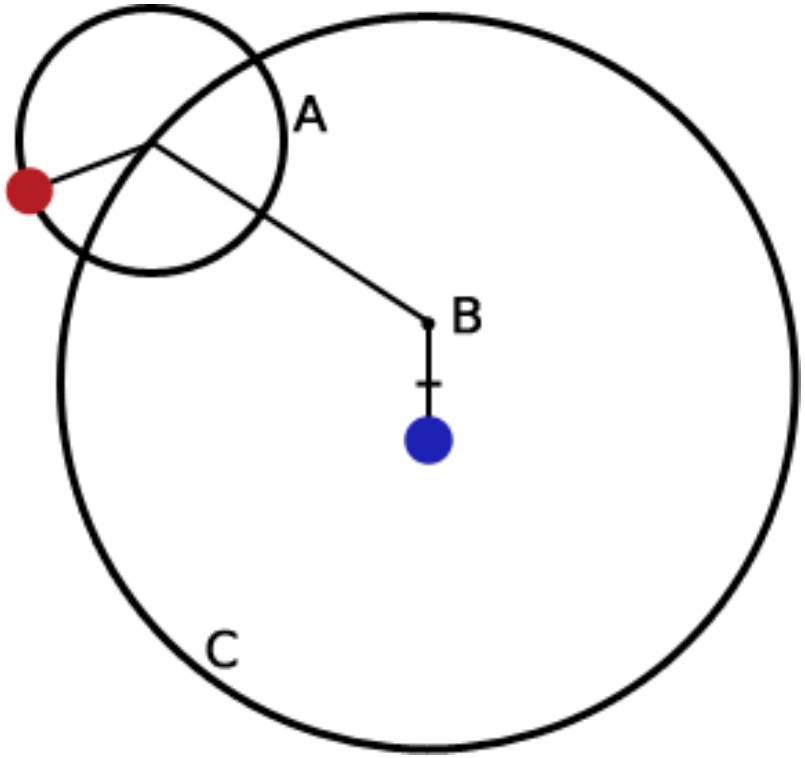
Ptolemy

Ptolemy

Ptolemy

Idea: More circles!

Epicycles



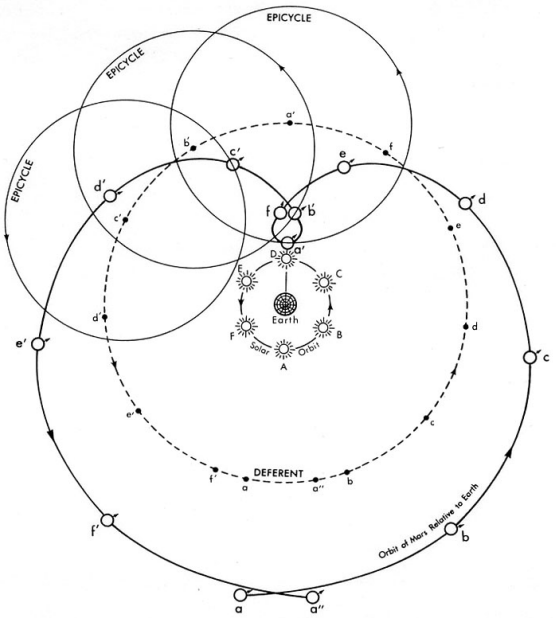
Epicycles

Not Bad!

Retrograde Motion of Mars

Not Bad!

Retrograde Motion of Mars



Epicyclic Motion

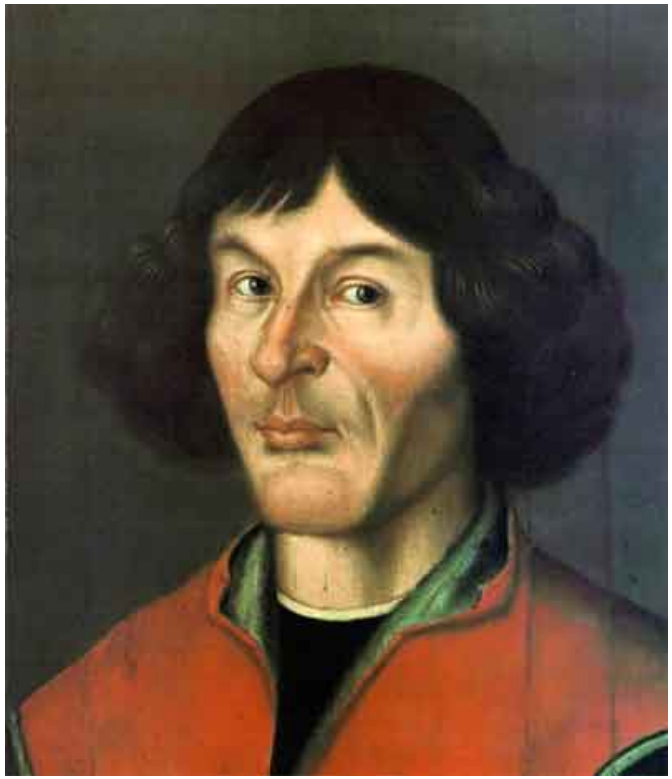
Not Bad!

- Ptolemy needs 39 epicycles.

Not Bad!

- Ptolemy needs 39 epicycles.
- But reasonably accurate 1000 years after publication, in 1504!
- Unclear that Ptolemy's goal was to accurately describe—just compute.

Copernicus



Copernicus

The Copernican System

- 48 epicycles, though they were smaller than Ptolemy's

The Copernican System

- 48 epicycles, though they were smaller than Ptolemy's
- Copernicus claims Ptolemy had 80 epicycles!

The Copernican System

- 48 epicycles, though they were smaller than Ptolemy's
- Copernicus claims Ptolemy had 80 epicycles!
- Serious problem—the lack of stellar parallax

Epicycles Get a Bad Rap

- An actual orbit:

$$x(t) = a \cdot \cos(\omega \cdot t), y(t) = b \cdot \sin(\omega \cdot t)$$

Epicycles Get a Bad Rap

- An actual orbit:

$$x(t) = a \cdot \cos(\omega \cdot t), y(t) = b \cdot \sin(\omega \cdot t)$$

- Circle if $a = b$

Epicycles Get a Bad Rap

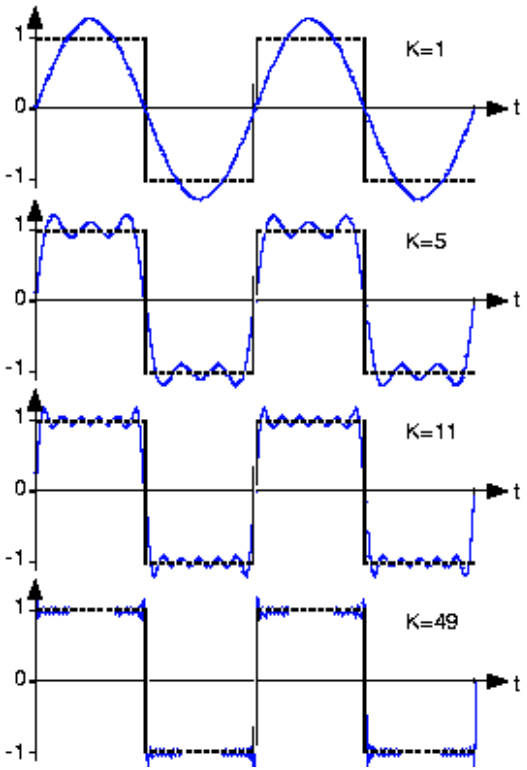
- An actual orbit:

$$x(t) = a \cdot \cos(\omega \cdot t), y(t) = b \cdot \sin(\omega \cdot t)$$

- Circle if $a = b$
- Fact (Fourier): Any periodic path can be written as a sum of circles!

$$x(t) = \sum a_i \cdot \cos(\omega_i \cdot t), y(t) = \sum a_i \cdot \sin(\omega_i \cdot t)$$

Epicycles Get a Bad Rap



Fourier Analysis

Kepler

Kepler

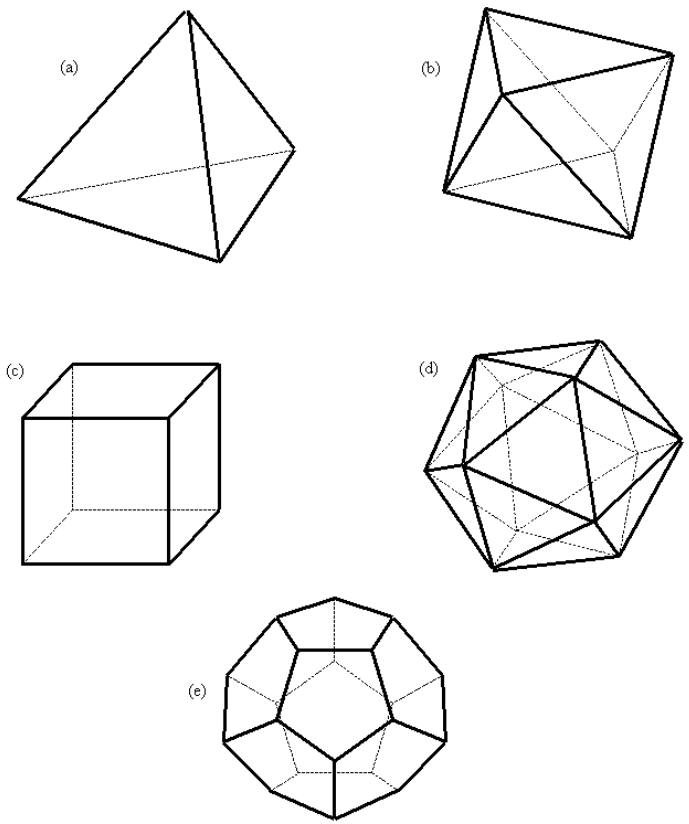
Kepler's First Question

Why are there only six planets? (Mercury, Venus, Earth, Mars, Jupiter, Saturn)

The Answer

Kepler's First Model of the Universe

The Platonic Solids



The Platonic Solids

The Platonic Solids

$$V - E + F = 2$$

The Platonic Solids

$$V - E + F = 2$$

$$aF = 2E = bV$$

where a is the number of edges per face and b is the number of edges per vertex

The Platonic Solids

$$V - E + F = 2$$

$$aF = 2E = bV$$

where a is the number of edges per face and b is the number of edges per vertex

Thus

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2} + \frac{1}{E} > \frac{1}{2}$$

The Platonic Solids

$$V - E + F = 2$$

$$aF = 2E = bV$$

where a is the number of edges per face and b is the number of edges per vertex

Thus

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2} + \frac{1}{E} > \frac{1}{2}$$

Only possibilities are

$$(a, b) = (3, 3), (3, 4), (4, 3), (5, 3), (3, 5).$$

Enter Tycho Brahe

Tycho Brahe

Kepler's Laws

Essentially by carefully studying the orbit of mars, Kepler postulated:

- The orbits of the planets are ellipses, with the sun at one focal point

Kepler's Laws

Essentially by carefully studying the orbit of mars, Kepler postulated:

- The orbits of the planets are ellipses, with the sun at one focal point
- The line joining a planet to the sun sweeps out equal areas in equal time

Kepler's Laws

Essentially by carefully studying the orbit of mars, Kepler postulated:

- The orbits of the planets are ellipses, with the sun at one focal point
- The line joining a planet to the sun sweeps out equal areas in equal time
- The square of the orbital period is proportional to the cube of the semi-major axis of the orbit.

Kepler's Laws

Essentially by carefully studying the orbit of mars, Kepler postulated:

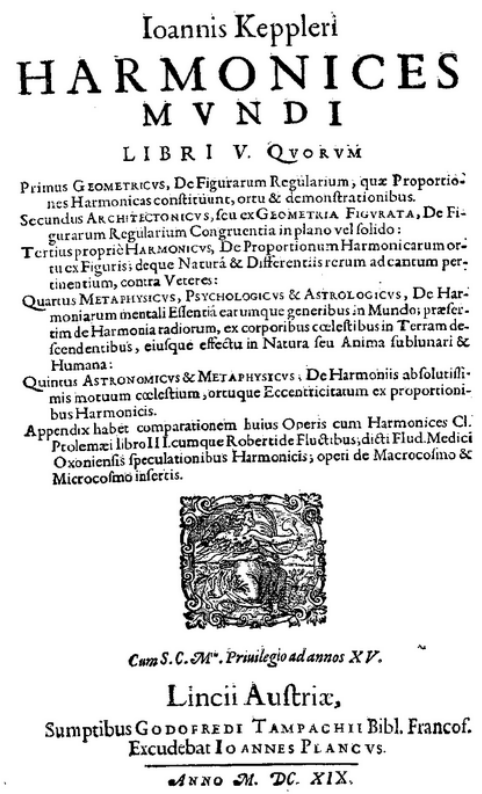
- The orbits of the planets are ellipses, with the sun at one focal point
- The line joining a planet to the sun sweeps out equal areas in equal time
- The square of the orbital period is proportional to the cube of the semi-major axis of the orbit.

Essentially extracted by Newton from a long and mystical tome of Kepler's.

Kepler's Actual Theory

A Baroque Construction

Kepler's Actual Theory



Ioannis Keppleri

HARMONICES MVNDI

LIBRI V. QVORVM

Primus GEOMETRICVS, De Figurarum Regularium, quæ Proportiones Harmonicas constituunt, ortu & demonstrationibus.
Secundus ARCHITECTONICVS, seu ex GEOMETRIA FIGVRATA, De Figurarum Regularium Congruentia in plano vel solido:
Tertius propriè HARMONICVS, De Proportionum Harmonicarum ortu ex Figuris; deque Naturâ & Differentiis rerum ad cantum pertinentium, contra Veteres:
Quartus METAPHYSICVS, PSYCHOLOGICVS & ASTROLOGICVS, De Harmoniarum mentali Essentiâ earumque generibus in Mundo; præsertim de Harmonia radiorum, ex corporibus cœlestibus in Terram descendentibus, eiusque effectu in Natura seu Anima sublunari & Humana:
Quintus ASTRONOMICVS & METAPHYSICVS, De Harmoniis absolutissimis motuum cœlestium, ortuque Eccentricitatum ex proportionibus Harmonicis.
Appendix habet comparationem huius Operis cum Harmonices Cl. Ptolemæi libro III. cumque Roberti de Fluctibus, dicti Flud. Medici Oxonienſis ſpeculationibus Harmonicis; operi de Macrocoſmo & Microcoſmo inſertis.

Cum S. C. M^t. Priuilegio ad annos XV.

Lincii Auſtriæ,
Sumptibus GODOFREDI TAMPACHII Bibl. Francof.
Excudebat IOANNES PLANCVS.
ANNO M. DC. XIX.

Music of the Spheres

The Modern Day

But why should we care about all this stumbling about in the dark? We've escaped our mystical obsessions in the modern day, right?

The Modern Day

But why should we care about all this stumbling about in the dark? We've escaped our mystical obsessions in the modern day, right?

Right?

The Modern Day

"The central idea of string theory is quite straightforward. If you examine any piece of matter ever more finely, at first you'll find molecules, atoms, sub-atomic particles. Probe the smaller particles, you'll find something else, a tiny vibrating filament of energy, a little tiny vibrating string." –Brian Greene

Exercises and Discussion

- How large is stellar parallax?

Exercises and Discussion

- How large is stellar parallax?
- How would you measure the circumference of the earth? Can you estimate it right now? What about the diameter?

Exercises and Discussion

- How large is stellar parallax?
- How would you measure the circumference of the earth? Can you estimate it right now? What about the diameter?
- How far is the earth from the sun? How would you measure the distance from the sun to another planet? From the earth to another planet?

Exercises and Discussion

- How large is stellar parallax?
- How would you measure the circumference of the earth? Can you estimate it right now? What about the diameter?
- How far is the earth from the sun? How would you measure the distance from the sun to another planet? From the earth to another planet?
- How can we tell the earth isn't flat? What shape is the earth?

Exercises and Discussion

- How large is stellar parallax?
- How would you measure the circumference of the earth? Can you estimate it right now? What about the diameter?
- How far is the earth from the sun? How would you measure the distance from the sun to another planet? From the earth to another planet?
- How can we tell the earth isn't flat? What shape is the earth?
- How can we tell that different celestial bodies are at different distances from the earth?

Exercises and Discussion

- What are some other plausible models of the solar system? How would the world look different if they were the truth (e.g. if the earth was flat)?

Exercises and Discussion

- What are some other plausible models of the solar system? How would the world look different if they were the truth (e.g. if the earth was flat)?
- What are some existing physical theories which are in conflict with one another? How might we decide which (if any) are correct?

Exercises and Discussion

- What are some other plausible models of the solar system? How would the world look different if they were the truth (e.g. if the earth was flat)?
- What are some existing physical theories which are in conflict with one another? How might we decide which (if any) are correct?
- Are any of our current physical beliefs based on mysticism or other non-scientific reasoning?